Investigation of AMOC in two GCMs using Linear Inverse Modeling

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Consider two models: CCSM4 and ESM2M

- Extensive analysis precludes a large number of models.

- These two models have very different AMOC behavior, so we look for commonalities and differences in variability and predictability.

- Region considered: Atlantic Ocean northward of 20S.

- Consider five fields as state variables (10 PCs per variable):
  - Ψ_{NoEk}, T_{upper}, T_{lower}, S_{upper}, S_{lower} (Heat Flux and Fresh Water Flux vary too rapidly)

Governing equation used for analysis:

…which is shorthand for

$$\frac{dx}{dt} = \mathbf{L}x + \xi$$

$$\frac{d}{dt}\begin{pmatrix} \Psi_{NoEk} \\ T_{upper} \\ T_{lower} \\ S_{upper} \\ S_{lower} \end{pmatrix} = \begin{pmatrix} L_{\Psi\Psi} & L_{\Psi Tup} & L_{\Psi Tlow} & L_{\Psi Sup} & L_{\Psi Slow} \\ L_{Tup\Psi} & L_{TupTup} & L_{TupTlow} & L_{TupSup} & L_{TupSlow} \\ L_{Tlow\Psi} & L_{TlowTup} & L_{TlowTlow} & L_{TlowSup} & L_{TlowSlow} \\ L_{Sup\Psi} & L_{SupTup} & L_{SupTlow} & L_{SupSup} & L_{SupSlow} \\ L_{Slow\Psi} & L_{SlowTup} & L_{SlowTlow} & L_{SlowSup} & L_{SlowSlow} \end{pmatrix}\begin{pmatrix} \Psi_{NoEk} \\ T_{upper} \\ T_{lower} \\ S_{upper} \\ S_{lower} \end{pmatrix} + \begin{pmatrix} \xi_{\Psi} \\ \xi_{Tup} \\ \xi_{Tlow} \\ \xi_{Sup} \\ \xi_{Slow} \end{pmatrix}$$

$$\text{Let } \mathbf{G}(\tau) = \exp(\mathbf{L}\tau) = \mathbf{1} + (\mathbf{L}\tau) + (\mathbf{L}\tau)^2/2! + (\mathbf{L}\tau)^3/3! + \ldots$$

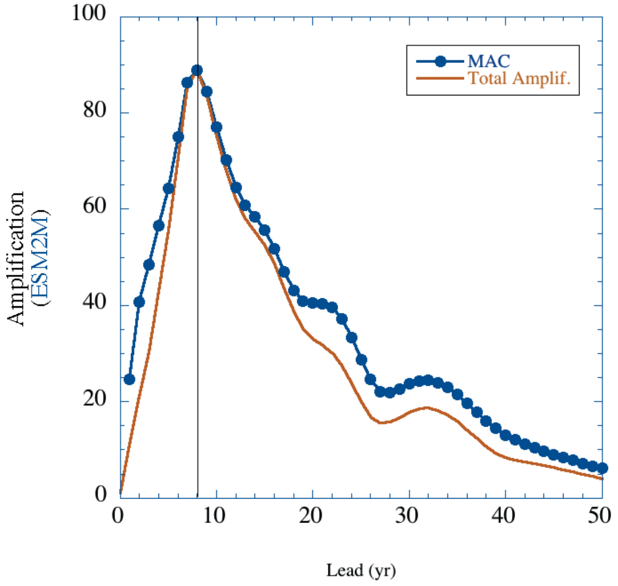
Given an initial condition $\boldsymbol{x}_o$, the predictable dynamics are just

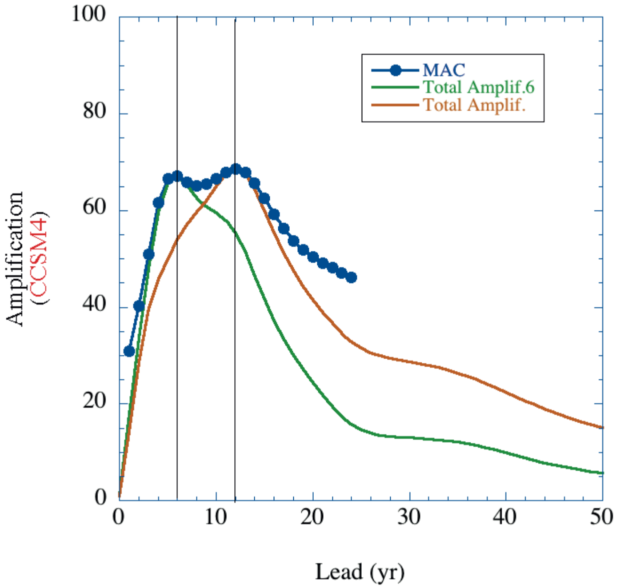
$$\boldsymbol{x}(t+\tau) = \mathbf{G}(\tau)\, \boldsymbol{x}_o(t).$$

Further, if $\mathbf{G}(\tau)$ is asymmetric, the initial condition $\boldsymbol{x}_o(t)$ giving the largest amount of growth at time $t+\tau$ is the right singular vector of $\mathbf{G}(\tau)$ (i.e., an eigenvector of $\mathbf{G}^T\mathbf{G}(\tau)$, called an "*optimal structure*") and the amplification factor λ is the corresponding eigenvalue.

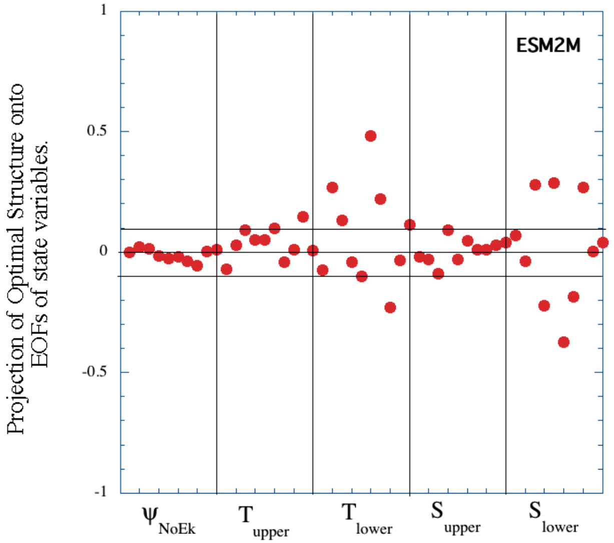
We estimate $\mathbf{G}(\tau)$ from the lagged covariance statistics of Ψ_{NoEk}, T_{upper}, T_{lower}, S_{upper}, and S_{lower} in a reduced space (10 PCs per variable).

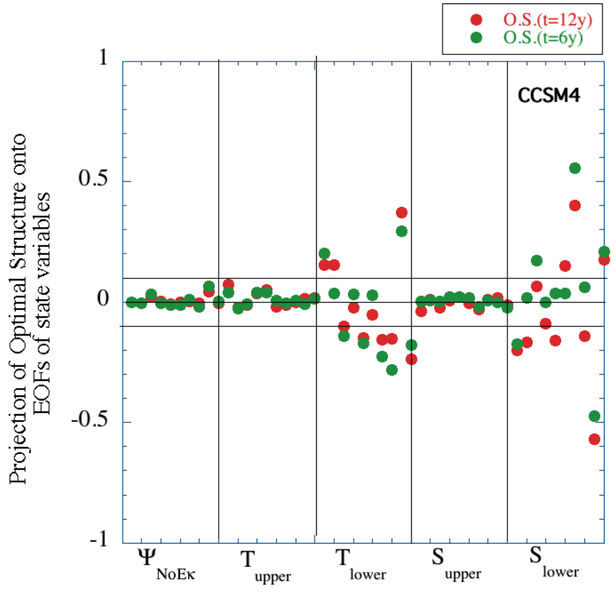
Right singular values:
(Eigenvalues of $\mathbf{G}^T\mathbf{G}(\tau)$)





Right singular vectors,
or “optimal structures”:
(Eigenvectors of $\mathbf{G}^T\mathbf{G}(\tau)$
for τ = 8 yrs, left, and
τ = 6 and 12 yrs, right)



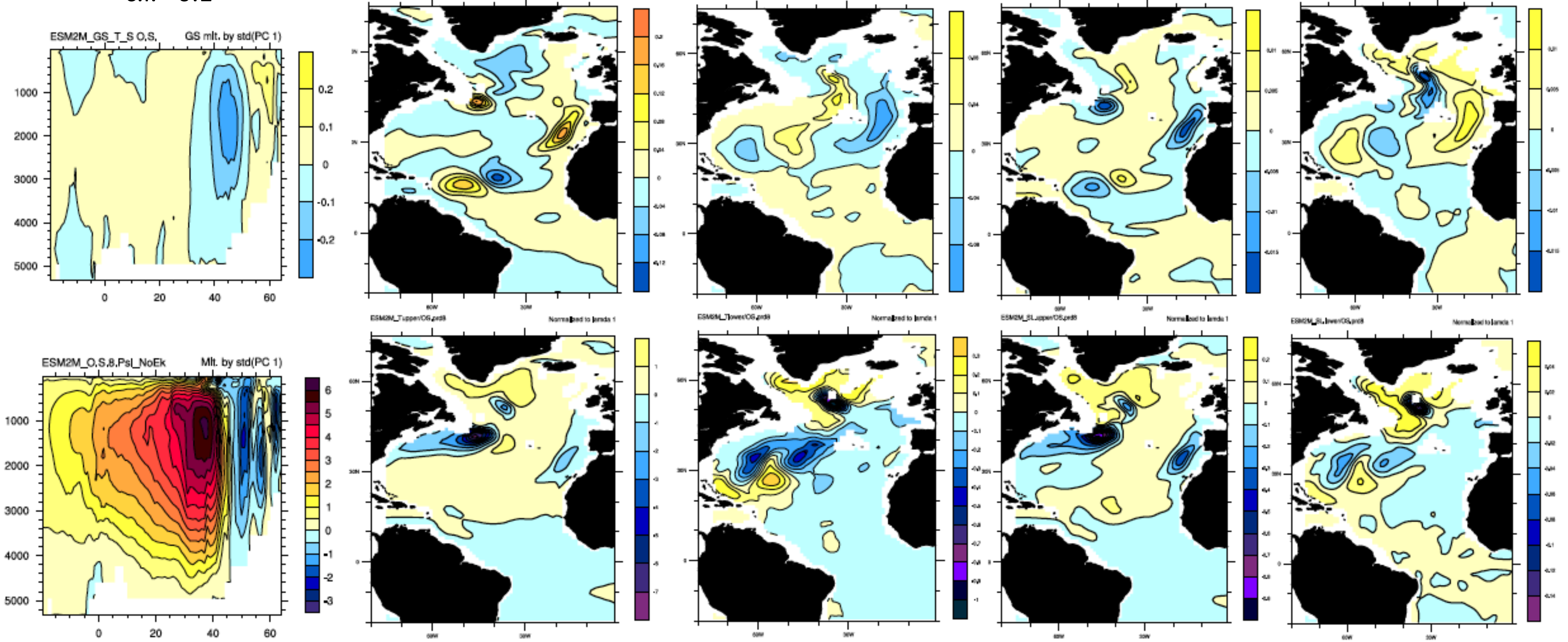


ESM2M:

Ψ_{NoEk} | T_{up} | T_{low} | S_{up} | S_{low}

This initial condition…. (RSV)

c.i. =0.1 | c.i. = 0.04 | c.i. = 0.04 | c.i. = .005 | c.i. = .005

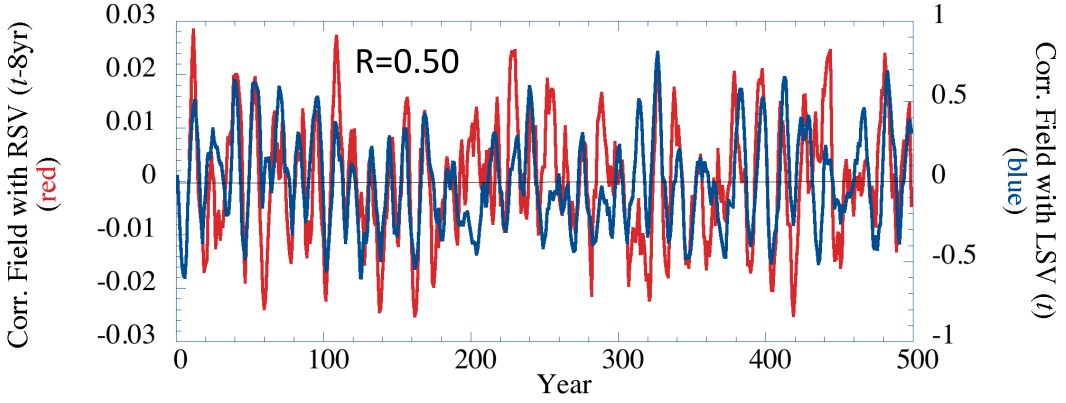


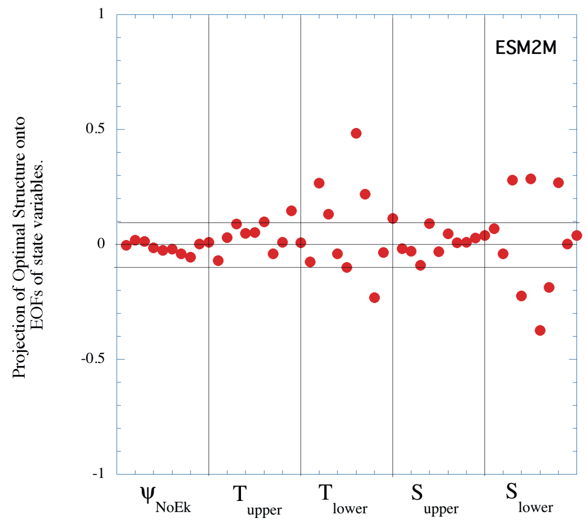
…evolves into this 8 years later. (LSV)

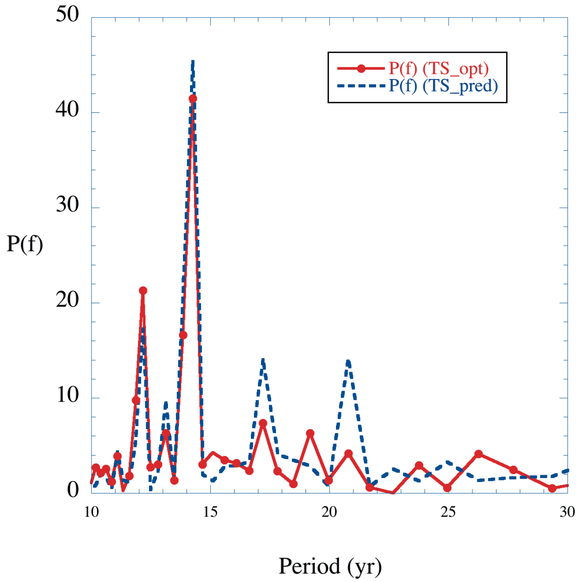
c.i. =0.5 | c.i. = 1.0 | c.i. = 0.1 | c.i. = 0.1 | c.i. = 0.02

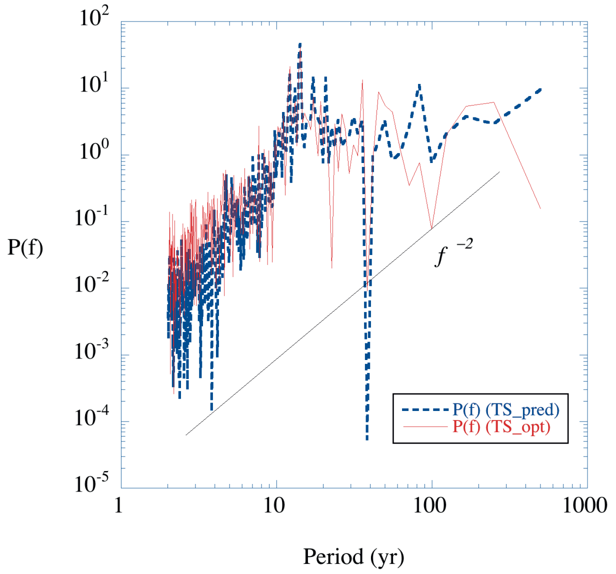
ESM2M (cont.)

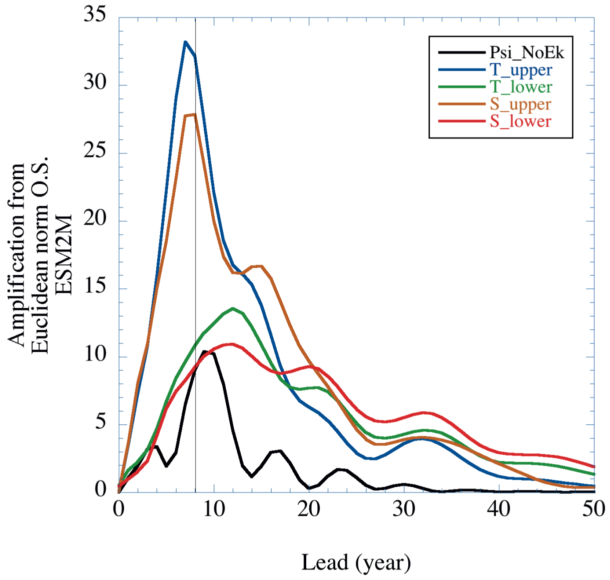
The 8-year prediction of AMOC in ESM2M would have a correlation skill score up to 0.5.







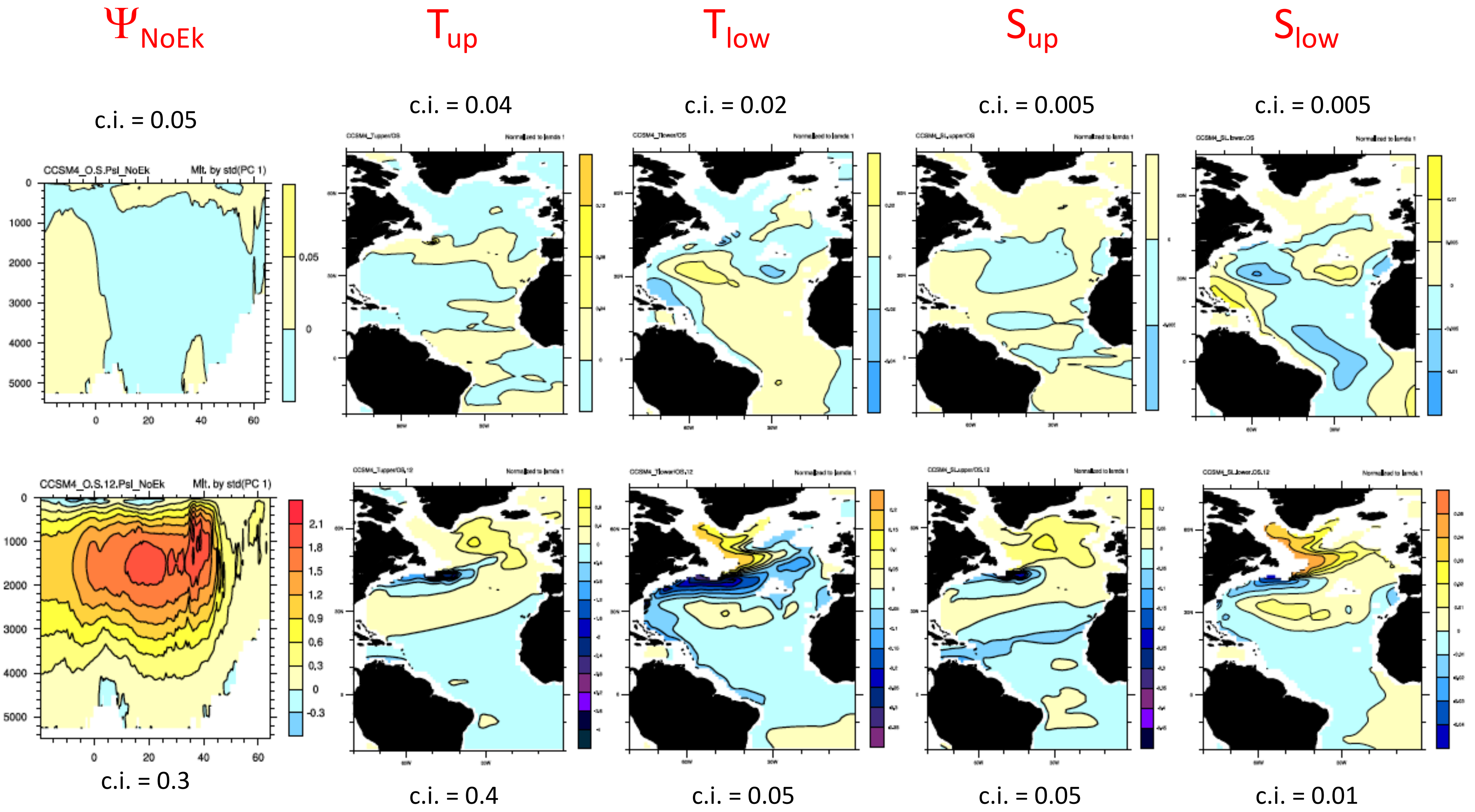




CCSM4:

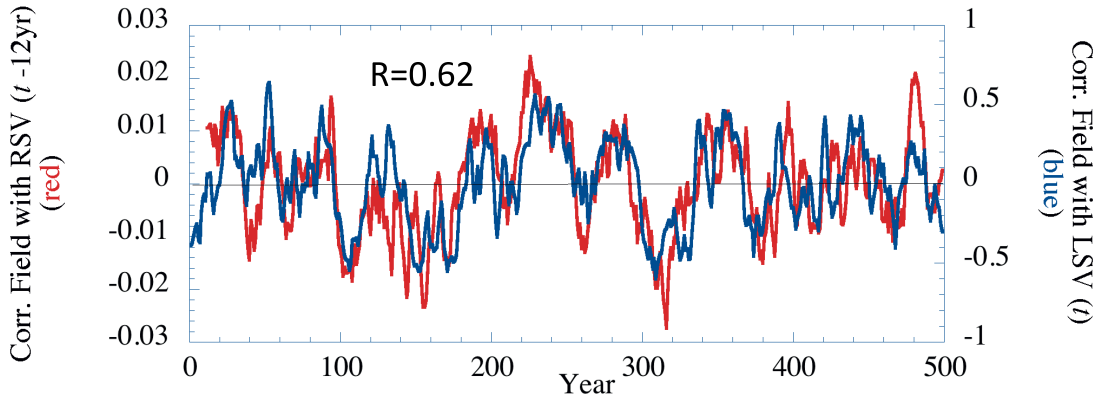
This initial condition…. (RSV)

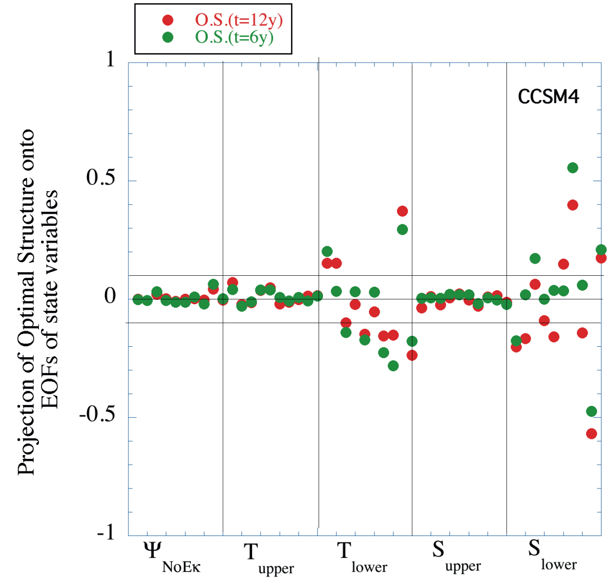
…evolves into this 12 years later. (LSV)

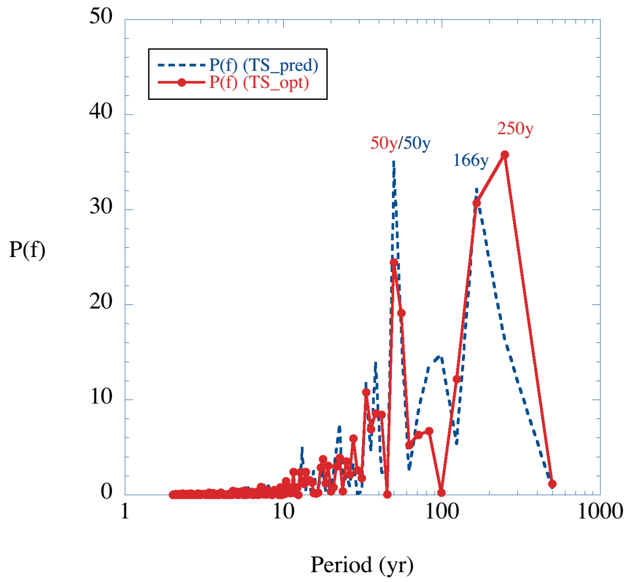


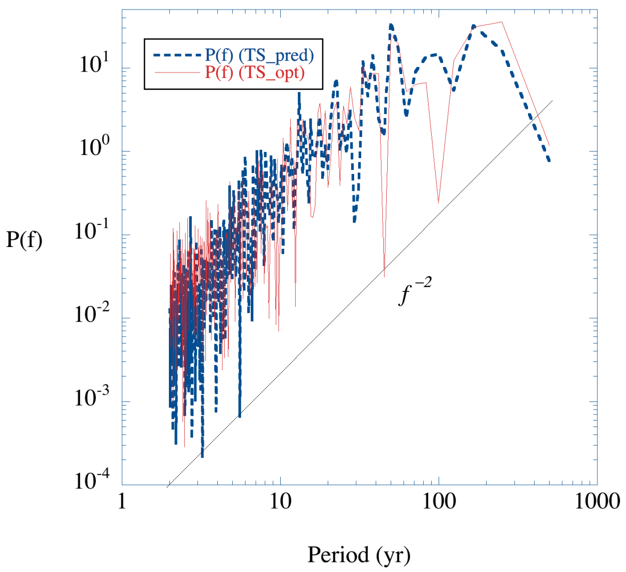
CCSM4(cont.)

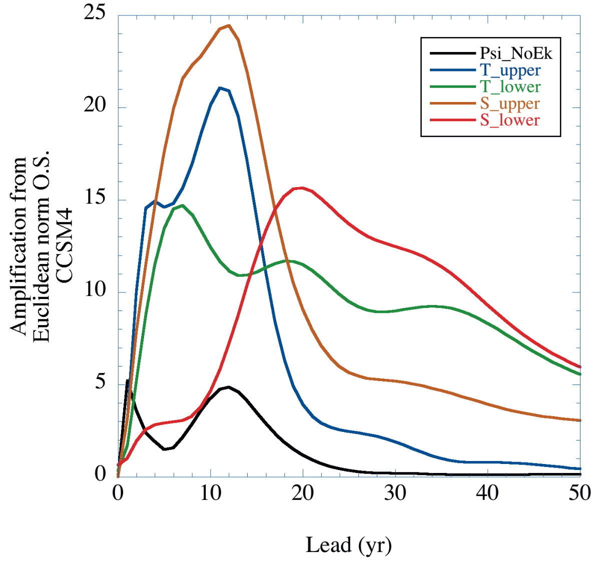
The 12-year prediction of AMOC in CCSM4 would have a correlation skill score up to 0.62.











Recall: $\frac{dx}{dt} = \mathbf{L}x + \xi$

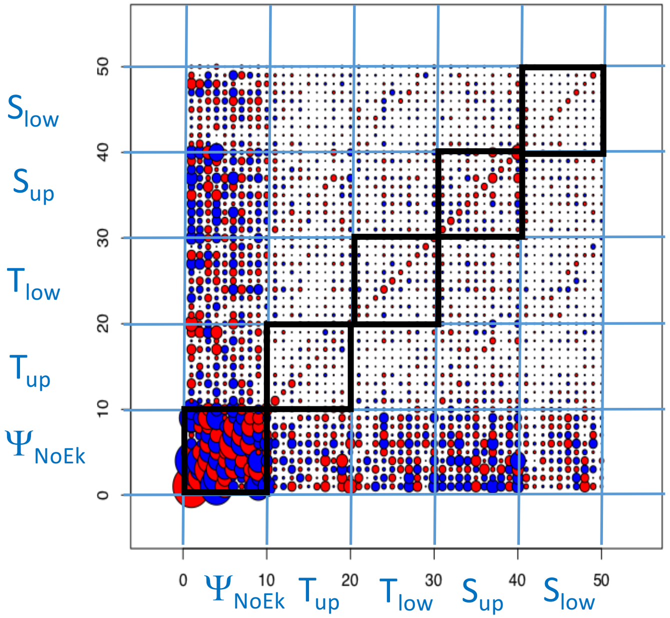
Covariance matrix $\mathbf{Q} = \langle \xi\xi^T \rangle dt$ of stochastic forcing

a) $\xi(t) = (x(t+\Delta) - x(t-\Delta))/2\Delta - \mathbf{L}x(t)$

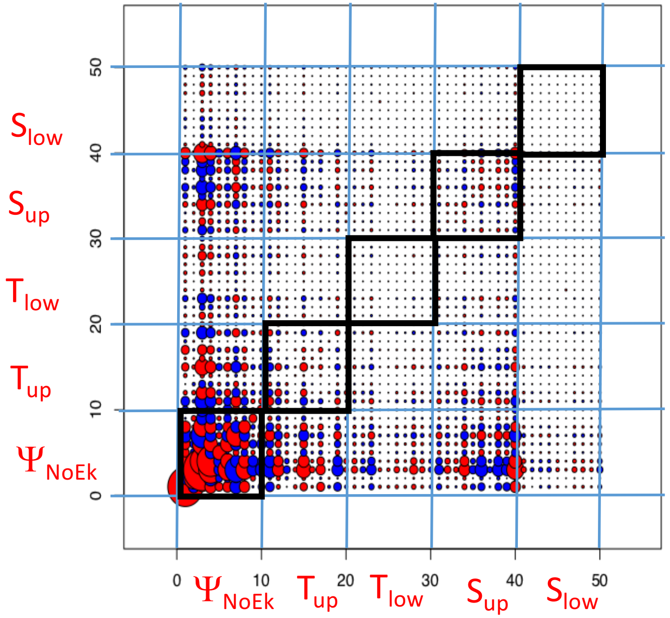
b) $-\mathbf{Q} = \mathbf{L}\langle x(t)x(t)^{\mathrm{T}} \rangle + \langle x(t)x(t)^{\mathrm{T}} \rangle \mathbf{L}^{\mathrm{T}}$

ξ_{ESM2M} is not highly correlated with either HF or FW

Q: ESM2M



Q: CCSM4



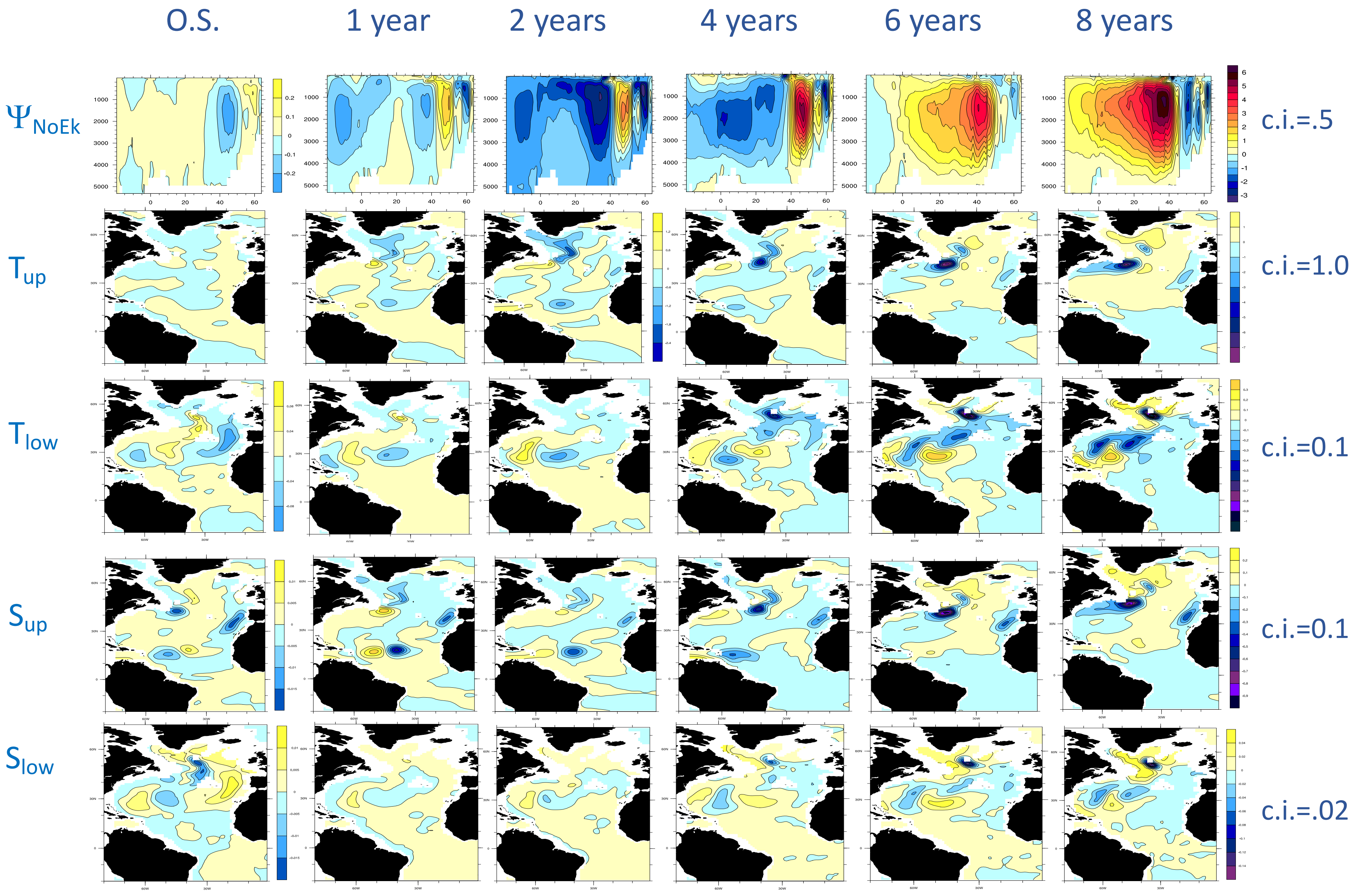
ξ_{CCSM4} has about 20% of its variance explained by HF and FW

Conclusions:

- The AMOC in ESM2M is much more periodic than that in CCSM4. The spectrum of AMOC is dominated by higher frequencies in ESM2M than in CCSM4. Still, AMOC in CCSM4 appears more predictable than in ESM2M.
- High amplification of AMOC in both models is dominated by characteristic low level patterns of temperature and salinity.
- The oscillation in ESM2M is likely to be nonlinear and self-sustained (no $\pi/2$ phase difference). The frequency of the oscillation is about 12 years.
- The spectrum of CCSM4 is dominated by lower frequencies (periods of 50-250 yrs).
- (Not shown) These results don't strongly depend on whether the Euclidean norm or the Ψ_{NoEk}– norm is used in estimating the left and right singular vectors.
- (Also not shown) Including annually-averaged fresh water FW and surface heat flux HF in the state vector gives inconsistent results because these variables vary faster than on the annual timescale.
- Stochastic forcing affects Ψ_{NoEk} more directly than it does the other state variables (T_{up}, T_{low}, S_{up}, S_{low}). S_{low}, the dominant predictor of AMOC in both models, does not appear to be stochastically forced.
- Monthly-scale FW and HF don't seem to provide much stochastic forcing to ESM2M; they provide about 20% of the stochastic forcing to CCSM4.
- The rapid (sub-annual) variability of Ψ_{NoEk} in CCSM4 appears to be real rather than an artifact of the analysis.

Sup. slide #1.

Linear prediction of evolution from the RSV to the LSV in 8 years. (ESM2M)



Sup. slide #2.

Linear prediction of evolution from the RSV to the LSV in 12 years. (CCSM4)